Any set that represents the value of the Regular Expression is called a **Regular Set.**

**Properties of Regular Sets**

**Property 1**. *The union of two regular set is regular.*

**Proof** −

Let us take two regular expressions

RE1 = aaaaa\* and RE2 = aaaa\*

So, L1 = {a, aaa, aaaaa,.....} StringsofoddlengthexcludingNullStringsofoddlengthexcludingNull

and L2 ={ ε, aa, aaaa, aaaaaa,.......} StringsofevenlengthincludingNullStringsofevenlengthincludingNull

L1 ∪ L2 = { ε, a, aa, aaa, aaaa, aaaaa, aaaaaa,.......}

StringsofallpossiblelengthsincludingNullStringsofallpossiblelengthsincludingNull

RE (L1 ∪ L2) = a\* whichisaregularexpressionitselfwhichisaregularexpressionitself

**Hence, proved.**

**Property 2.** *The intersection of two regular set is regular.*

**Proof** −

Let us take two regular expressions

RE1 = aa∗a∗ and RE2 = aaaa\*

So, L1 = { a,aa, aaa, aaaa, ....} StringsofallpossiblelengthsexcludingNullStringsofallpossiblelengthsexcludingNull

L2 = { ε, aa, aaaa, aaaaaa,.......} StringsofevenlengthincludingNullStringsofevenlengthincludingNull

L1 ∩ L2 = { aa, aaaa, aaaaaa,.......} StringsofevenlengthexcludingNullStringsofevenlengthexcludingNull

RE (L1 ∩ L2) = aaaaaa\* which is a regular expression itself.

**Hence, proved.**

**Property 3.** *The complement of a regular set is regular.*

**Proof** −

Let us take a regular expression −

RE = aaaa\*

So, L = {ε, aa, aaaa, aaaaaa, .......} StringsofevenlengthincludingNullStringsofevenlengthincludingNull

Complement of **L** is all the strings that is not in **L**.

So, L’ = {a, aaa, aaaaa, .....} StringsofoddlengthexcludingNullStringsofoddlengthexcludingNull

RE L′L′ = aaaaa\* which is a regular expression itself.

**Hence, proved.**

**Property 4.** *The difference of two regular set is regular.*

**Proof** −

Let us take two regular expressions −

RE1 = a a∗a∗ and RE2 = aaaa\*

So, L1 = {a, aa, aaa, aaaa, ....} StringsofallpossiblelengthsexcludingNullStringsofallpossiblelengthsexcludingNull

L2 = { ε, aa, aaaa, aaaaaa,.......} StringsofevenlengthincludingNullStringsofevenlengthincludingNull

L1 – L2 = {a, aaa, aaaaa, aaaaaaa, ....}

StringsofalloddlengthsexcludingNullStringsofalloddlengthsexcludingNull

RE (L1 – L2) = a aaaa\* which is a regular expression.

**Hence, proved.**

**Property 5.** *The reversal of a regular set is regular.*

**Proof** −

We have to prove **LR** is also regular if **L** is a regular set.

Let, L = {01, 10, 11, 10}

RE LL = 01 + 10 + 11 + 10

LR = {10, 01, 11, 01}

RE (LR) = 01 + 10 + 11 + 10 which is regular

**Hence, proved.**

**Property 6.** *The closure of a regular set is regular.*

**Proof** −

If L = {a, aaa, aaaaa, .......} StringsofoddlengthexcludingNullStringsofoddlengthexcludingNull

i.e., RE LL = a aaaa\*

L\* = {a, aa, aaa, aaaa , aaaaa,……………} StringsofalllengthsexcludingNullStringsofalllengthsexcludingNull

RE L∗L∗ = a aa\*

**Hence, proved.**

**Property 7.** *The concatenation of two regular sets is regular.*

**Proof −**

Let RE1 = 0+10+1\*0 and RE2 = 010+10+1\*

Here, L1 = {0, 00, 10, 000, 010, ......} Setofstringsendingin0Setofstringsendingin0

and L2 = {01, 010,011,.....} Setofstringsbeginningwith01Setofstringsbeginningwith01

Then, L1 L2 = {001,0010,0011,0001,00010,00011,1001,10010,.............}

Set of strings containing 001 as a substring which can be represented by an RE − 0+10+1\*0010+10+1\*

Hence, proved.

**Identities Related to Regular Expressions**

Given R, P, L, Q as regular expressions, the following identities hold −

* ∅\* = ε
* ε\* = ε
* RR\* = R\*R
* R\*R\* = R\*
* R∗R∗\* = R\*
* RR\* = R\*R
* (PQPQ)\*P =P(QPQP)\*
* a+ba+b\* = a∗b∗a∗b∗\* = a∗+b∗a∗+b∗\* = a+b∗a+b∗\* = a\*ba∗ba∗\*
* R + ∅ = ∅ + R = R TheidentityforunionTheidentityforunion
* R ε = ε R = R TheidentityforconcatenationTheidentityforconcatenation
* ∅ L = L ∅ = ∅ TheannihilatorforconcatenationTheannihilatorforconcatenation
* R + R = R IdempotentlawIdempotentlaw
* L M+NM+N = LM + LN LeftdistributivelawLeftdistributivelaw
* M+NM+N L = LM + LN RightdistributivelawRightdistributivelaw
* ε + RR\* = ε + R\*R = R\*